Strong $M1$ Transitions in Light Nuclei*

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The nature of strong $M1$ transitions, particularly those observed by inelastic electron scattering at backward angles, is investigated. Calculations of particular cases reveal a tendency to concentrate the transition strength in a few levels, in some respects similar to the familiar giant $E1$ resonance. The energy-weighted sum rule for such transitions is examined, and an approximation is developed which exhibits the qualitative behavior to be expected in $4N$ nuclei whose ground states have $I=0=T$. The results are compared with experiment.

I. INTRODUCTION

EXCITATION of the nucleus by inelastic scattering of electrons has provided much information about nuclear properties. The particular experiments toward which this discussion is directed are those[1] in which the scattered electrons are observed at large backward angles. These experiments, which are designed to investigate magnetic transitions, select states which have large matrix elements for transitions from the ground state. The variation of intensity with electron energy often permits a determination of the multipolarity, and a number of prominent $M1$ transitions connected to the ground states of light nuclei have been observed. A particularly advantageous feature of excitation by electrons is that many of these transitions are not seen as gamma decays because the levels are unstable to nucleon emission, a much more probable mode of decay.

The operator for $M1$ transitions can be written as the sum of two contributions, one of which is a scalar in isobaric-spin space and the other a vector. In the scalar part the neutron and proton spin contributions tend to cancel, so that $M1$ transitions between $T=0$ states (for which only the scalar part contributes) tend to be quite weak—a feature pointed out by Morpurgo.[2] On the other hand, the neutron and proton spin contributions are additive in the vector part of the $M1$ operator which can lead to strong transitions. This treatment will be confined to nuclei whose ground states have isobaric spin $T=0$, so that strong $M1$ transitions to $T=1$ states are caused by the isobaric vector operator whose z component is

$$\mu_z=(\mu_n-\mu_p)\sum_k s_z(k)\tau_3(k)-\tfrac{1}{2}\sum_k l_z(k)\tau_3(k). \quad (1)$$

The problem is investigated theoretically by use of nuclear wave functions obtained from the Hamiltonian

$$\mathcal{H}=\sum_k\mathcal{H}_0(k)+a\sum_k\mathbf{l}(k)\cdot\mathbf{s}(k)+\sum_{i>k}V(i,k), \quad (2)$$

which is the usual sort for a shell model with spin-orbit coupling and two-body interactions of a central-force nature. In the first section specific examples from the $1p$ shell are treated and the results are compared with experiment. Next, general features are pointed out and a sum rule is evaluated. The latter has particular usefulness for $4N$ nuclei. General behavior beyond the $1p$ shell is then extrapolated in terms of the sum rule.

II. PARTICULAR CASES

This section contains the results of calculations for those $1p$-shell nuclei whose ground states have $T=0$. The wave functions for these nuclei are taken from an early calculation[3] which used a harmonic-oscillator form of $\mathcal{H}_0(k)$ in Eq. (2) and a particular two-body interaction for $V(i,k)$ that fits the low-lying levels of the energy spectrum for Li^6. Since the $M1$ operator does not contain any radial dependence, matrix elements are insensitive to the radial wave functions. Furthermore, it has been shown that the wave functions of many low-lying states of the $1p$ shell can be obtained via a generating procedure[4] without reference to $V(i,k)$. This latter result indicates that the wave functions have a widespread validity such that a large class of $V(i,k)$ lead to very similar wave functions for these low-lying states. The correlations in these wave functions are probably determined by the predominant attractive even-state interactions, a feature resulting from fitting the two-nucleon spectrum of Li^6 and common to the various popular forms of $V(i,k)$.

Therefore, the general features that appear in the calculations of $M1$ transitions are not likely to be sensitive to the particular $V(i,k)$. Instead, they reflect the more basic correlations present in the wave functions. The quantity best suited for comparison with experiment is the reduced transition probability, $B(M1;I_0T_0\rightarrow IT)$. This is related to the transition width Γ by an energy-dependent factor, namely,

$$\Gamma=2.76\times10^{-3}E^3B,$$

where the units of Γ are eV and the energy is in MeV.

A. $4N$ Nuclei

There are two $4N$ nuclei in the $1p$ shell which are not closed shells, namely Be^8 and C^{12}. Their ground

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[1] W. C. Barber, F. Berthold, G. Fricke, and F. E. Gudden, Phys. Rev. **120**, 2081 (1960). For an excellent review article, see W. C. Barber in Ann. Rev. Nucl. Sci. **12**, 1 (1962).
[2] G. Morpurgo, Phys. Rev. **110**, 721 (1958).
[3] D. Kurath, Phys. Rev. **101**, 216 (1956); **106**, 975 (1957).
[4] D. Kurath and L. Pičman, Nucl. Phys. **10**, 313 (1959).

states have $I=0$ so that only $I=1=T$ states can be reached by the isobaric vector $M1$ operator. There are eight $I=1=T$ states that can be formed with four $1p$ nucleons (or holes). Calculated reduced transition probabilities $B(M1;00\rightarrow 11)$ for transitions from the ground state to the four lower $I=1=T$ states in each nucleus are given in Table I as a function of the relative strength[5] (a/K) of spin-orbit coupling. The table also shows that over 99% of the transition strength is contained in transitions to the lower four of the eight possible transitions in each case.

TABLE I. Reduced transition probabilities $B(M1)$ for magnetic-dipole transitions in Be^8 and C^{12} from the ground state $(I=0=T)$ to various excited states having $I=1=T$. The states are labeled by ν, starting with the lowest excitation energy. Values of $B(M1)$ are listed as functions of the spin-orbit coupling parameter a/K. The percentage of the total $B(M1)$ from the ground state which is contained in the listed transitions is also given.

	ν \ a/K	1.5	3.0	4.5	6.0
	1	1.90	3.76	5.30	6.63
	2	0.55	0.00	0.33	1.01
Be^8	3	0.88	0.81	0.72	0.73
	4	0.21	0.28	0.41	0.56
		99.9%	99.7%	99.6%	99.6%
	1	1.28	4.36	9.97	17.26
	2	1.04	0.70	0.44	0.25
C^{12}	3	1.12	0.66	0.25	0.03
	4	0.20	0.21	0.23	0.28
		99.8%	99.4%	99.3%	99.8%

Moreover a large fraction of the strength is concentrated in the transition to the lowest state, especially for large values of a/K. These particular transitions have been discussed previously[3] and compared with experimentally observed gamma widths from the 17.6-MeV state of Be^8 and the 15.1-MeV state of C^{12}. They are very sensitive to the value of the parameter (a/K), and provide about the only information about this parameter in $4N$ nuclei.

The states at higher excitation are unstable against particle emission so gamma decay is not seen. They are also missed in inelastic electron scattering, presumably because of their comparatively weak transition strengths.

B. $(4N+2)$ Nuclei

The simplest $1p$-shell nuclei in this category are Li^6 and N^{14}. The ground states of both Li^6 and N^{14} have $I=1$ so that strong $M1$ transitions are possible to states with $I=0$, 1, or 2 and $T=1$. Because the configurations are so simple there is a total of only five states with $T=1$ for each nucleus, including two each for $I=0$ and 2.

In Li^6 about 90% of the transition strength $B(M1;10\rightarrow I1)$ is calculated to lie in the transition to the lower $I=0$, $T=1$ state, with a numerical value $B(M1)\approx 23$ which is very insensitive to (a/K). From the measured width[6] of this transition, which is observed in gamma decay, one can extract $B(M1)=(24\pm 5)$. An independent measurement of this transition by inelastic electron scattering[1] gives $B(M1)=(16\pm 2)$. For low (a/K) most of the remaining strength is calculated to lie in the transition to the upper $I=2$, $T=1$ state at about 10 MeV.

The $M1$ transitions in N^{14} have been treated thoroughly by Warburton and Pinkston.[7] A calculation restricted to the $(1p)^{-2}$ configuration shows that some 90% of the $B(M1;10\rightarrow 11)$ is concentrated in the transition to the lower $I=2$, $T=1$ state calculated to lie at about 10 MeV. Again the strength is quite insensitive to (a/K) and has the value $B(M1)\approx 19$. Experimental observation of gamma transitions indicates that this strength is split between two states[8] at 9.2 and 10.4 MeV with $B(M1)$ values of 7 and 8, respectively. These two transitions are the only $M1$ transitions seen in inelastic electron scattering and have strengths consistent with those seen in gamma decay. Warburton and Pinkston interpret this splitting of the transition strength to mean that the (2,1) state from the $(1p)^{-2}$ configuration is mixed with a (2,1) state from the $(1p)^{-4}(2s,1d)^2$ configuration, something one would expect at such excitation energies and near the end of the $1p$ shell.

The remaining stable odd-odd nucleus in the $1p$ shell is B^{10} which lies in the middle of the shell and contains many states arising from the $(1p)^6$ configuration. The ground state has $I=3$, $T=0$ so that $M1$ transitions to $T=1$ states can occur for $I=2$, 3, or 4. Reduced transition probabilities for such transitions together with rough calculated excitation energies are presented in Table II. A noteworthy feature is that the transition strength is again concentrated into the low-lying levels. Although there are 14 states with $I=2$, 95% of the strength lies in the three lowest levels. A similar concentration occurs for the 7 levels with $I=3$ and the 4 levels with $I=4$. Again the strengths are mostly insensitive to variation of (a/K) in the range of physical interest.

The experimental information from $M1$ gamma decay from $T=1$ levels to the ground state indicates[10] that the transition from the $I=2$ level at 5.16 MeV is quite weak. The results of inelastic electron scattering[9] show three prominent $M1$ transitions at energies of 7.9, 11.8, and 14.0 MeV. The extracted reduced transition probabilities $B(M1;3\rightarrow I)$ for these peaks are $(8.8\pm 20\%)$, $(6.2\pm 50\%)$, and $(3.1\pm 50\%)$, respectively. A comparison with Table II indicates possible identification of the

[5] In (a/K) the quantity K is a representative integral of the two-body interaction.

[6] L. Cohen and R. A. Tobin, Nucl. Phys. **14**, 243 (1959).

[7] E. K. Warburton and W. T. Pinkston, Phys. Rev. **118**, 733 (1960).

[8] H. J. Rose, Nucl. Phys. **19**, 113 (1960). For a discussion of $M1$ strengths, see E. K. Warburton, in *Electromagnetic Lifetimes and Properties of Nuclear States* (National Research Council–National Academy of Sciences, Washington, D. C., 1962), Nuclear Science Series 37, Publication No. 974, p. 180.

[9] R. D. Edge and G. A. Peterson, Phys. Rev. **128**, 2750 (1962).

[10] L. Meyer-Schützmeister and S. S. Hanna, Phys. Rev. **108**, 1506 (1957); E. L. Sprenkel, J. W. Olness, and R. E. Segel, Phys. Rev. Letters **7**, 174 (1961).

TABLE II. Reduced transition probability $B(M1)$ for magnetic-dipole transitions from the ground state ($I=3$, $T=0$) of B^{10} to various excited states having $T=1$. The states and their approximate calculated excitation energies are identified on the left. Values of $B(M1)$ are given as functions of the relative strength (a/K) of spin-orbit coupling. The percentage of total $B(M1; 3\rightarrow I)$ which is concentrated in the listed transitions is also given.

I	E (MeV) \ a/K	3.0	4.5	6.0
2	5	0.85	0.06	0.11
2	7.5	12.98	11.66	11.00
2	12.5	2.06	2.78	2.91
		94%	95%	96%
3	10.5	4.99	6.39	7.07
3	16	0.08	0.68	1.59
		90%	92%	95%
4	12	0.40	1.03	1.63
4	16	0.06	0.03	0.02
		80%	90%	93%

7.9-MeV transition as the one to the second calculated ($I=2$, $T=1$) state, the 11.8-MeV transition as the one going to the lowest ($I=3$, $T=1$) state, and the 14.0-MeV transition as involving either the third ($I=2$, $T=1$) state or the lowest ($I=4$, $T=1$) state (or both states since this is a broad peak). Such identifications are very useful in locating levels from the $(1p)^6$ configuration since this is the major configuration of the B^{10} ground state and these strong $M1$ transitions in electron scattering arise through such components of the wave functions.

III. APPLICATION OF THE SUM RULE

A. Formulation

In order to extrapolate beyond the $1p$ shell, it is useful to study the energy-weighted sum rule, which demonstrates explicitly some of the general features found in the particular examples. The expectation value[11] in the ground state of the double commutator between the operator μ_z of Eq. (1) and the Hamiltonian $\mathcal{H}$ of Eq. (2) is found to be

$$2\sum_n(E_n-E_0)|\langle n|\mu_z|0\rangle|^2=\langle 0|[\mu_z,[\mathcal{H},\mu_z]]|0\rangle, \quad (3)$$

where the summation is over all possible states. The left-hand side of Eq. (3) can be expressed in terms of the reduced transition probabilities $B(M1)$ by putting in the angular-momentum quantum numbers and summing over magnetic quantum numbers with the help of the Eckart-Wigner theorem. The effect of this summation on the right-hand side of Eq. (3) is to limit the expectation value to that part of the commutator which is scalar in ordinary space. Since this discussion is limited to ground states with isobaric spin $T=0$, all the excited states, n, will have $T=1$ so the isobaric spin indices can be suppressed, and Eq. (3) becomes

$$\sum_{I,\nu}[E(I,\nu)-E(I_0)]B(M1;I_0\rightarrow I,\nu) = \tfrac{3}{2}\langle I_0|[\mu_z,[\mathcal{H},\mu_z]]_{\text{scalar}}|I_0\rangle. \quad (4)$$

In order for this sum to be useful there must be enough concentration of transition strength in the low levels to overcome the energy weighting and permit experimental evaluation of the left-hand side of Eq. (4). Then a theoretical evaluation of the right-hand side of Eq. (4) will indicate the sort of behavior to expect. Such an application is clearly limited in that the weak transitions will often be overlooked. But in view of the concentration tendencies evident in the particular cases, one may hope to obtain a rough rule. The $4N$ nuclei provide the most favorable conditions for application of the sum rule. This is because most of the transition strength appears to be concentrated in the lower $I=1$ levels, which, however, lie at fairly high excitation (above 15 MeV in the $1p$ shell). It will also be shown that evaluation of the commutator leads to an expression which is especially simple for the $4N$ nuclei.

B. Test for $4N$ Nuclei

In order to test the usefulness of Eq. (4), again consider the Be^8 and C^{12} examples. Experimentally only one $M1$ transition from the ground state is positively identified in each nucleus, namely the transition to the lowest $I=1=T$ state. The observed strengths agree with the calculated values for $(a/K)\approx 3$ in Be^8 and $(a/K)\approx 4.5-6$ in C^{12}. In order to get an indication of how much of the sum in Eq. (4) is represented by the observed transition, the sum was evaluated by using computed strengths and energies for the higher excited states. This procedure should at least give the order of magnitude of these contributions. The results are shown graphically for Be^8 in Fig. 1 and for C^{12} in Fig. 2 for several values of the parameter a/K. The individual contributions to Eq. (4) from the lower four levels are

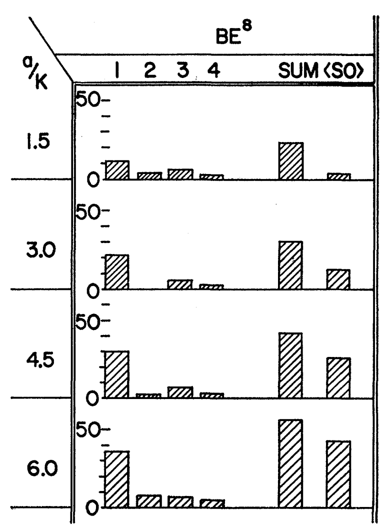

FIG. 1. Contributions to the sum on the left-hand side of Eq. (4) from the four lowest calculated $I=1=T$ states of Be^8 for different values of a/K. The complete sum is also given, and the last column gives the contribution to the right-hand side of Eq. (4) arising from the spin-orbit term of the Hamiltonian.

[11] For this general approach to sum rules, see R. G. Sachs and N. Austern, Phys. Rev. **81**, 705 (1951).

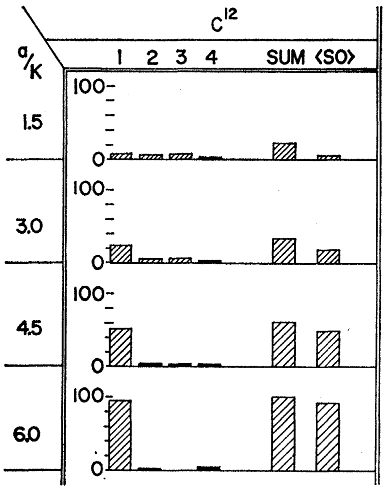

FIG. 2. Contributions to the sum on the left-hand side of Eq. (4) from the four lowest calculated $I=1=T$ states of C^{12} for different values of a/K. The complete sum is also given, and the last column gives the contribution to the right-hand side of Eq. (4) arising from the spin-orbit term of the Hamiltonian.

given together with the total sum for all eight levels. Even with the energy weighting, the contribution from the upper four levels is negligible. Two other points to note are (1) the sensitivity of the transition strength to a/K and (2) the fact that for the larger values of a/K the sum for C^{12} is about double that for Be^8, as indicated by the need for different scales.

The right-hand side of Eq. (4) can be evaluated by assuming the Hamiltonian of Eq. (2) and forming the double commutator between it and the μ_z of Eq. (1). The $\mathcal{H}_0$ term commutes with μ_z so it does not contribute. The commutator with the spin-orbit term provides a simple result in that the scalar part of this commutator is just

$$-\tfrac{2}{3}(\mu_n-\mu_p+\tfrac{1}{2})^2[a\sum_k \mathbf{l}(k)\cdot\mathbf{s}(k)].$$

Therefore this contribution to the right-hand side of Eq. (4) is directly proportional to the ground-state expectation value of the spin-orbit coupling term of $\mathcal{H}$. This contribution is given in the last columns of the tables in Figs. 1 and 2 under the heading ⟨SO⟩.

Comparison of this contribution with the sum which was evaluated theoretically leads to the conclusion that for $a/K=4.5$ and 6 the spin-orbit contribution dominates the ground-state expectation value of the commutator. This occurs even though such values of a/K are still far from the value for jj coupling—as shown by the fact that the ground-state expectation values of $\mathbf{l}\cdot\mathbf{s}$ in this region are only 50–60% of the maxima attained at the jj limit. The dominance of the spin-orbit contribution explicitly demonstrates the sensitivity to a/K of the sum in Eq. (4). It also explains that for a given $a/K\geqslant 4.5$ the sum for C^{12} is about twice as large as the sum for Be^8 simply because there are twice as many $1p$ nucleons present.

C. The Two-Body Interaction

There remains the contribution of $V(i,k)$ to the right-hand side of Eq. (4). To be consistent one should use the form of $V(i,k)$ which was chosen in calculating the nuclear wave functions. However, because of the wider validity of the wave functions discussed in the first part of Sec. II, it is instructive to look at the general central interactions before specializing. The general central interaction[12] is

$$V=F(r_{ik})[\mathcal{S}(\tau)\mathcal{T}(\sigma)+{}^{31}A\,\mathcal{T}(\tau)\mathcal{S}(\sigma)+{}^{11}A\,\mathcal{S}(\tau)\mathcal{S}(\sigma)+{}^{33}A\,\mathcal{T}(\tau)\mathcal{T}(\sigma)], \quad (5)$$

where $F(r_{ik})$ contains the radial dependence of the central force, while $\mathcal{S}$ and $\mathcal{T}$ are the singlet and triplet projection operators, respectively, for the indicated coordinates. They are defined by

$$4\mathcal{S}(\sigma)=4[1-\mathcal{T}(\sigma)]=1-\boldsymbol{\sigma}(i)\cdot\boldsymbol{\sigma}(k). \quad (6)$$

Different exchange mixtures are given by choosing the numerical coefficients A, and while the Li^6 spectrum requires ${}^{31}A\approx 0.6$, information about the other coefficients is meager since it comes chiefly from higher excited states about which little is known.

Because of the large coefficient $(\mu_n-\mu_p)$, the spin-dependent part of μ_z would be expected to provide the largest contribution to the commutator between μ_z and $V(i,k)$. This contribution comes only from the spin-dependent part of $V(i,k)$. The contribution to the right-hand side of Eq. (4) from just the spin-dependent part of μ_z is then

$$(\mu_n-\mu_p)^2\{(1-{}^{31}A)\langle I_0|\sum F(r_{ik})\times[\mathcal{T}(\tau)\mathcal{S}(\sigma)-\mathcal{S}(\tau)\mathcal{T}(\sigma)]|I_0\rangle+\tfrac{1}{3}({}^{33}A-{}^{11}A)\times\langle I_0|\sum F(r_{ik})[9\mathcal{S}(\tau)\mathcal{S}(\sigma)-\mathcal{T}(\tau)\mathcal{T}(\sigma)]|I_0\rangle\}. \quad (7)$$

For $4N$ nuclei this contribution tends to be very small in comparison with the contribution from the spin-orbit term. In the LS limit of negligible spin-orbit coupling, the relative numbers of singlet and triplet couplings in the $I_0=0$ ground states of the $4N$ nuclei are determined solely by the statistical weights of these couplings. Hence, both the expectation values in expression (7) vanish. For the $V(i,k)$ used in the Be^8 and C^{12} calculations, the contribution from expression (7) is less than 10% of the contribution from the spin-orbit-coupling term for all values of a/K. This very probably is true for any $V(i,k)$ which does not have an abnormally large spin dependence.

The small values of expression (7) for Be^8 and C^{12} means that the difference between the columns labeled SUM and ⟨SO⟩ in Figs. 1 and 2 arises from the orbital part of μ_z in Eq. (1), and that its contribution is not very sensitive to a/K. This difference has not been evaluated explicitly.

For $(4N+2)$ nuclei, the contribution (7) will not be small since even in the LS limit the singlet and triplet coupling will not cancel out in the wave function for the odd-odd ground state. For example, the Li^6 ground state is an eigenfunction of $\mathcal{S}(\tau)\mathcal{T}(\sigma)$ with unit eigenvalue in the LS limit, and expression (7) makes a large contribu-

[12] L. Rosenfeld, *Nuclear Forces* (North-Holland Publishing Company, Amsterdam, 1948), p. 160.

tion. This will lead to sums which are not very sensitive to a/K, in contrast to the case for $4N$ nuclei.

D. Extrapolation to the $(2s-1d)$ Shell

The insight obtained from analysis of the $1p$ shell enables one to make a reasonable extrapolation to the $(2s-1d)$ shell. The $M1$ strength should again be concentrated in a few low levels. Furthermore, strong $M1$ transitions in $4N$ nuclei should arise when the spin-orbit term dominates the right-hand side of Eq. (4). A semiquantitative approximation, valid for $4N$ nuclei in which the transitions are from an $I=0=T$ ground state to $I=1=T$ excited states, is then

$$\sum_\nu (E_\nu - E_0) B(M1; 0\rightarrow \nu) \doteq -a(\mu_n-\mu_p+\tfrac{1}{2})^2 \langle 0|\sum_k \mathbf{l}(k)\cdot\mathbf{s}(k)|0\rangle. \quad (8)$$

Since the $2s$ nucleons give zero contribution to the expectation value in Eq. (8), that value depends only on the degree of filling of the $1d_{5/2}$ and $1d_{3/2}$ levels. The right-hand side of Eq. (8) would have its maximum value if the $1d_{5/2}$ level were full and the $1d_{3/2}$ level were empty. Therefore, as one proceeds through the $(2s-1d)$ shell, the sum for the $M1$ transitions is expected to build up from very weak transitions at the beginning of the shell to strong transitions in the region of Si^{28} and S^{32} where presumably the condition of a filled $1d_{5/2}$ level and an empty $1d_{3/2}$ level is most closely approached. Then the strength should drop quickly as the $1d_{3/2}$ shell is filled. This behavior agrees qualitatively with what is found[13] by inelastic electron scattering.

When more detailed experimental results are available, they can be interpreted in terms of the filling of $1d_{5/2}$ and $1d_{3/2}$ levels in Eq. (8). For this purpose one requires wave functions for the ground state. The simplest approach in such a semiquantitative procedure is to estimate the expectation value of $\sum \mathbf{l}\cdot\mathbf{s}$ by using wave functions χ_0 consisting of groups of 4 nucleons in Nilsson[14] levels. This leads to different values of the expectation value of $\sum \mathbf{l}\cdot\mathbf{s}$ depending on the levels and the value of Nilsson's parameter η. An interesting possibility occurs for Si^{28}, where for oblate deformation with $\eta=-4$ and with levels Nos. 5, 6, and 7 filled, one gets $\langle \mathbf{l}\cdot\mathbf{s}\rangle = 8.0$; for prolate deformation with $\eta=+4$ and levels Nos. 6, 7, and 9 filled, one gets $\langle \mathbf{l}\cdot\mathbf{s}\rangle = 4.3$. Thus one may be able to decide which deformation is appropriate. It may eventually be worth the effort to see how far the model can be pushed by including the full commutator of Eq. (4) together with ground-state wave functions for which angular momentum is a good quantum number rather than the χ_0 approximation.

[13] W. C. Barber, J. Goldemberg, G. A. Peterson, and Y. Torizuka, Stanford Report HEPL-276 (1962) (unpublished).
[14] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **29**, No. 16 (1955).

IV. CONCLUSIONS

The most striking feature which arises from the analysis of strong $M1$ transitions from ground states in the $1p$ shell is the tendency to concentrate the strength in a few transitions. Just as for the giant $E1$ resonance one can say that the $T=1$ magnetic-dipole state, $\mu_z\psi_0$, is localized. But whereas the $E1$ resonance is concentrated in the high-excitation states of a given I, the $M1$ resonance is concentrated in the low-excitation states of a given I.

For $4N$ nuclei with $T=0$, the energy-weighted sum rule provides an indication of where to expect strong $M1$ transitions. This semiquantitative rule is appropriate to the experiments on inelastic electron scattering, in which the strong $M1$ transitions from the ground state are prominent. There seems to be at least qualitative agreement with observation in the region of the $(2s-1d)$ shell.

In order to attempt a more nearly quantitative application of the sum rule, one would require ground-state wave functions. These can be obtained either via projection[4] from Nilsson many-nucleon functions or by doing intermediate-coupling calculations. The interaction $V(i,k)$ should also be included in the commutator with μ_z. For $(4N+2)$ nuclei it might appear that expression (7) can lead to some restrictions on the interaction coefficients ^{31}A, ^{11}A, and ^{33}A. However, only ^{31}A which is already quite well determined is likely to be important, because for these nuclei the expectation value multiplying $(1-{}^{31}A)$ should be much larger than that multiplying $({}^{33}A-{}^{11}A)$. Eventually an accurate evaluation of the sum rule would require consideration of small admixtures to the pure shell-model configurations that were assumed. Because of the energy weighting, their effect may be magnified; but it is likely to be a higher order effect for these light nuclei.

For odd-A nuclei whose ground states have $T=\frac{1}{2}$ the operator μ_z of Eq. (1) is still likely to provide the strong transitions. However, one can now have transitions to other $T=\frac{1}{2}$ levels as well as $T=\frac{3}{2}$ levels so the effect of concentration is probably much less pronounced.

At any rate, the simple features of the sum rule are most appropriate for the $4N$ nuclei whose ground states have $I=0=T$. In these nuclei one can interpret the observations qualitatively in terms of the model Hamiltonian of Eq. (2) and the operator μ_z of Eq. (1). Whether more refined comparison is possible remains to be seen.

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